

$D0 - D4$ system and QCD_{3+1}

Kenji Suzuki

Institute of Physics, University of Tokyo, Komaba,

Tokyo 153, Japan

ksuzuki@hep1.c.u-tokyo.ac.jp

Abstract

We consider a $(3 + 1)$ -dimensional QCD model using a dual supergravity description with a non-extremal $D0$ - $D4$ brane background. We calculate the spectrum of glueball masses and Wilson loops in the background. The mass spectrum is shown to coincide with one in non-extremal $D4$ -brane systems, and an area law of spatial Wilson loops is established. We show that there is a region that Kaluza-Klein modes of the Euclidean time direction are decoupled without decoupling glueball masses.

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I. INTRODUCTION

Recently years, Maldacena's conjecture [1,2], which is a duality between supersymmetric gauge theory in the large N strong-coupling limit and string theory on AdS backgrounds, has been discussed. For instance, the relations between correlation functions in gauge theory and effective actions in string theory on AdS backgrounds are established [3,4]. According to a method discussed in [5], Wilson loops for the large N gauge theories for the strong coupling limit can be obtained by calculating the interaction energy between the massive quark and the anti-quark separated by a distance L in terms of string theory on AdS backgrounds. The gauge theories are in a deconfinement phase.

Witten [6] extended the duality to the non-supersymmetric theories at zero temperature, which are obtained by the compactification of the Euclidean time direction with anti-periodic boundary conditions to fermions. The compactification radius R is given by $T = 1/2\pi R$, where T is the Hawking temperature in the supergravity description. The order of the fermion masses is T , and then supersymmetries are broken. In non-extremal D -brane backgrounds with these boundary conditions, the spectra of glueball masses have been calculated by solving the wave equations of the dilatons in supergravity description [7–10]. We observe an area law for spatial Wilson loops, which indicates that the large N gauge theories at zero temperature are in a confining phase as expected. The mass spectra are agreement with the lattice calculations [11–13]. However, there are some problems that Kaluza-Klein states cannot be decoupled without decoupling glueball masses in these models. In order to avoid the difficulty, the QCD models using rotating D -brane backgrounds have been discussed [14–18]. In these backgrounds, KK modes in the Euclidean time direction are decoupled without decoupling glueball masses in the limit that the angular momenta are infinite. There are still some unwanted KK states that do not decouple.

In this paper we study a $(3+1)$ -dimensional QCD model using the supergravity description of non-extremal $D0$ - $D4$ branes, in order to obtain the QCD model without KK-modes. The composition of intersecting D -branes in supergravity is discussed in [19–26]. We calculate the Wilson loop according to the method [5], and the glueball mass spectrum obtained by solving the wave equation of the dilaton in the background. In the case that the dilaton ϕ in the limit $r \rightarrow \infty$ is finite, we can not obtain the QCD model without KK modes. However, we show that in the case that the dilaton in the limit $r \rightarrow \infty$ vanishes, there is a region where KK modes of the Euclidean time direction are decoupled without decoupling glueball masses. Furthermore, the spatial Wilson loop exhibits a confining area law behavior, and the glueball mass spectrum is coincident with one in the non-extremal $D4$ -brane background.

The organization of this paper is as follows. In section 2, we calculate glueball masses using the supergravity approach with non-extremal $D0$ - $D4$ brane background. In section 3, we consider the spatial Wilson loops and compare glueball masses and Kaluza-Klein masses. In section 4, we consider in the case of the background that in the limit $r \rightarrow \infty$ the dilaton vanishes.

II. GLUEBALL MASSES

We consider glueball masses using the supergravity description, which are obtained by solving the wave equation of the dilaton in the supergravity background. We treat the background corresponding to the non-extremal $D0$ - $D4$ branes given by

$$ds^2 = f(r)^{-1/2}g(r)^{-1/2} \left[-h(r)dt^2 + g(r)\{dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2\} + f(r)g(r)\{h(r)^{-1}dr^2 + r^2d\Omega^2\} \right], \quad (1)$$

where

$$f(r) = 1 + \frac{g\alpha'Q_1}{r^3}, \quad g(r) = 1 + \frac{g\alpha'Q_2}{vr^3}, \quad h(r) = 1 - \frac{r_0^3}{r^3}, \quad (2)$$

with a dilaton background

$$e^{-2\phi(r)} = f(r)^{1/2}g(r)^{-3/2} = \sqrt{\frac{v^3r^6(r^3 + g\alpha'Q_1)}{(vr^3 + g\alpha'Q_2)^3}}, \quad (3)$$

which is finite in the limit $r \rightarrow \infty$. We consider the metric with the Euclidean time coordinate and $x_1 \rightarrow ix_1$, and we take the limit

$$U = \frac{r}{\alpha'} = \text{fixed}, \quad \alpha' \rightarrow 0. \quad (4)$$

The wave equation of the dilaton is

$$\partial_\mu e^{-2\phi} \sqrt{g} g^{\mu\nu} \partial_\nu \phi = 0. \quad (5)$$

Assuming that $\phi = e^{ikx_1} \rho(U)$, the wave equation reduces to

$$\partial_U (U^3 - U_T^3) U \partial_U \rho + M^2 f(U) U^4 \rho = 0, \quad (6)$$

where $M^2 = -k^2$. We denote that the equation is independent of the function $g(U)$, and the wave equation in this background is coincident with one in the non-extremal $D4$ -brane background. In the limit $\alpha \rightarrow 0$, the equation is

$$\partial_U (U^3 - U_T^3) U \partial_U \rho + M^2 g Q_1 U \rho = 0, \quad (7)$$

with $U_T = r_0/\alpha'$. Defining a new variable $x = U^2$ and rescaling, this equation reduces further to

$$\partial_x (x^{2+1/2} - x) \partial_x \rho + \sigma \rho = 0, \quad (8)$$

with $\sigma = M^2 g Q_1 / 4 U_T$. Using the WKB approximation [13], the mass spectrum is

$$M^2 = 16\pi^3 \frac{\Gamma(\frac{2}{3})}{\Gamma(\frac{1}{6})} \left(\frac{3}{8\pi}\right)^2 \frac{4U_T}{gQ_1} m(m+2) + O(m^0). \quad (m = 1, 2, 3, \dots) \quad (9)$$

If we take into account the dilaton fluctuations [27], there are some corrections due to the $D0$ -branes.

III. WILSON LOOP

We consider the Wilson loop for the full $D0$ - $D4$ brane background discussed in the previous section, in the region of the small but nonzero α' , because we need the small curvature in the regions of the large but finite Q_1, Q_2 . According to the method [5], the expectation value of the Wilson loop is

$$\langle W(C) \rangle \sim e^{-TE(L)} \sim e^{-S}, \quad (10)$$

where S is the Nambu-Goto action of a fundamental string. C denotes a closed loop in the $x_1 - x_2$ directions. L is the distance between the quark and the antiquark.

We consider the spatial Wilson loop in the $x_1 - x_2$ directions with Euclidean time coordinate and $x_1 \rightarrow -ix_1$. We take the Euclidean time coordinate as the space-like circle with the radius $R = 1/2\pi T$, where T is the Hawking temperature in the supergravity description. The fermions obey antiperiodic boundary conditions. In the low energy, we can obtain the zero temperature $(3+1)$ -dimensional QCD model. The world-sheet action in the $x_1 - x_2$ directions is

$$\begin{aligned} S_{NG} &= \frac{1}{2\pi\alpha'} \int dx_1 dx_2 \sqrt{\det G_{MN} \partial_\alpha X^M \partial_\beta X^N} \\ &= \frac{Y}{2\pi\alpha'} \int dy \sqrt{g(U)h(U)^{-1} \left(\frac{dU}{dy}\right)^2 + g(U)f(U)^{-1}} \\ &= \frac{Y}{2\pi\alpha'} \int dy \sqrt{\frac{\alpha'^2 U^3 v + gQ_2}{(U^3 - U_T^3)v} \left(\frac{dU}{dy}\right)^2 + \frac{\alpha'^2 U^3 v + gQ_2}{(\alpha'^2 U^3 + gQ_1)v}}, \end{aligned} \quad (11)$$

with $U_T = r_0/\alpha'$, and $y \equiv x_2$. Y is a period in the x_1 -direction. G_{MN} is the string metric of the non-extremal $D0$ - $D4$ brane background discussed in the previous section. The distance between the quark and the antiquark is

$$\begin{aligned} L = 2 \int dy &= 2\sqrt{g(U_0)/f(U_0)} \int_{U_0}^{\infty} dU \sqrt{\frac{g(U)f(U)}{g(U)h(U)(g(U)/f(U) - g(U_0)/f(U_0))}} \\ &= 2\frac{1}{\alpha'\sqrt{gQ_1v - gQ_2}} \int_{U_0}^{\infty} dU \sqrt{\frac{(\alpha'^2 U^3 + gQ_1)^2 (\alpha'^2 U_0^3 v + gQ_2)}{(U^3 - U_T^3)(U^3 - U_0^3)}} \\ &= 2\frac{1}{\alpha'\sqrt{gQ_1v - gQ_2}U_0^4} \int_1^{\infty} dx \sqrt{\frac{(\alpha'^2 x^3 U_0^3 + gQ_1)^2 (\alpha'^2 U_0^3 v + gQ_2)}{(x^3 - \lambda^3)(x^3 - 1)}} \end{aligned} \quad (12)$$

$$\rightarrow 2\frac{gQ_1\sqrt{U_0^3v + gq_2}}{\sqrt{gQ_1v - gQ_2}U_0^4} \int_1^{\infty} dx \frac{1}{\sqrt{(x^3 - \lambda^3)(x^3 - 1)}}, \quad (\alpha' \rightarrow 0) \quad (13)$$

where $x = U/U_0$ and $\lambda = U_T^3/U_0^3$. U_0 denotes the lowest value of U , and q_2 is defined by $Q_2 = \alpha'^2 q_2$. The energy is

$$E_{q\bar{q}} = \frac{1}{\alpha'\pi} \int_{U_0}^{\infty} dU \left[\sqrt{\frac{g(U)^2/f(U)}{h(U)(g(U)/f(U) - g(U_0)/f(U_0))}} - 1 \right] - \frac{1}{\pi} \int_{U_T}^{U_0} dU \sqrt{g(U)/h(U)}$$

$$\begin{aligned}
&= \frac{1}{\pi \alpha'^2 \sqrt{v(gQ_1v - gQ_2)}} \int_{U_0}^{\infty} dU \left[\sqrt{\frac{(\alpha'^2 U_0^3 + gQ_1)(\alpha'^2 U^3v + gQ_2)^2}{(U^3 - U_T^3)(U^3 - U_0^3)}} - 1 \right] + \frac{U_T - U_0}{\pi} \\
&= \frac{1}{\pi \alpha'^2 \sqrt{v(gQ_1v - gQ_2)} U_0^4} \int_1^{\infty} dx \left[\sqrt{\frac{(\alpha'^2 x^3 U_0^3 + gQ_1)(\alpha' x^3 U_0^3v + gQ_2)}{(x^3 - \lambda^3)(x^3 - 1)}} - 1 \right] + \frac{U_T - U_0}{\pi} \\
&\rightarrow \frac{\sqrt{gQ_1}}{\pi \sqrt{v(gQ_1v - gQ_2)} U_0^4} \int_1^{\infty} dx \left[\sqrt{\frac{(x^3 U_0^3v + gq_2)^2}{(x^3 - \lambda^3)(x^3 - 1)}} - 1 \right] + \frac{U_T - U_0}{\pi}. \quad (\alpha' \rightarrow 0) \quad (14)
\end{aligned}$$

We consider the large L behavior, which is obtained in the limit $\lambda \rightarrow 1$. In this limit, the main contribution to the integrals of L and E comes from the region near $x = 1$. Therefore we obtain the string tension as

$$T_{TM} = E_{q\bar{q}}/L = \frac{\sqrt{\alpha'^2 U_0^3v + gQ_2}}{2\alpha'\pi\sqrt{\alpha'^2 U_0^3v + gQ_1v}} \quad (15)$$

$$\rightarrow \sqrt{\frac{U_T^3}{4\pi^2 gQ_1}} \quad (gq_2 \ll U_0^3v) \quad (16)$$

$$\rightarrow \sqrt{\frac{q_2}{4\pi^2 Q_1v}}. \quad (gq_2 \gg U_0^3v) \quad (17)$$

The glueball mass is $M_{GB} \sim \sqrt{U_T/gQ_1}$, as discussed in the previous section. Kaluza-Klein masses are proportional to $1/R$, where $R = 1/T$ is the compactification radius of the time coordinate. The Hawking temperature T in the supergravity description is given by

$$T = \frac{3\alpha' U_T^2 \sqrt{v}}{4\pi \sqrt{(\alpha'^2 U_T^3 + gQ_1)(\alpha'^2 U_T^3v + gQ_2)}} \quad (18)$$

$$\rightarrow \frac{3\sqrt{U_T}}{4\pi \sqrt{gQ_1}} \quad (gq_2 \ll U_0^3v) \quad (19)$$

$$\rightarrow \frac{3U_T^2 \sqrt{v}}{4\pi g \sqrt{Q_1 q_2}}. \quad (gq_2 \gg U_0^3v) \quad (20)$$

In the limit $\alpha' \rightarrow 0$, KK masses are $M_{KK} \sim U_T^2 \sqrt{v}/\sqrt{gQ_1(U_0^3v + gq_2)}$. Then the ratio of the masses is

$$M_{KK}/M_{GB} = \sqrt{U_T^3v/(U_T^3v + gq_2)} \quad (21)$$

$$\rightarrow 1 \quad (gq_2 \ll U_T^3v) \quad (22)$$

$$\rightarrow \sqrt{U_T^3/gq_2} \ll 1. \quad (gq_2 \gg U_T^3v) \quad (23)$$

Therefore we can not obtain the region that KK masses in the Euclidean time direction are decoupled without decoupling glueball masses. We consider the way to resolve this problem in the next section.

IV. BACKGROUND WITH ZERO DILATON AT BOUNDARY

We consider the boundary of the non-extremal $D0$ - $D4$ brane background discussed in previous section. The background is the Minkowski space at $r \rightarrow \infty$, and there is no boundary. In the supergravity - SYM correspondence [2], it is needed that the background has a boundary at $r \rightarrow \infty$. In addition, we need a QCD model that KK masses are decoupled without decoupling glueball masses. In order to resolve their problems, we replace the harmonic function with

$$g(r) = 1 + \frac{g\alpha'Q_2}{vr^3} \rightarrow g'(r) = \frac{g\alpha'Q_2}{vr^3}, \quad (24)$$

which corresponds to

$$U_T^3 v + gq_2 \rightarrow gq_2, \quad (25)$$

in the equation of the ratio between glueball masses and KK masses (21). Then we can take the region $M_{KK}/M_{GB} \gg 1$. We note that this procedure is not to take the near horizon limit, but to replace the harmonic function in order to obtain the zero dilaton at the boundary. This means that the effects of the dilaton for Kaluza-Klein masses in the Euclidean time direction are suppressed at the boundary. Then we consider the metric given by

$$ds^2 = f(r)^{-1/2} g'(r)^{-1/2} \left[-h(r) dt^2 + g'(r) \{dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2\} + f(r) g'(r) \{h(r)^{-1} dr^2 + r^2 d\Omega^2\} \right], \quad (26)$$

where

$$f(r) = 1 + \frac{g\alpha'Q_1}{r^3}, \quad g'(r) = \frac{g\alpha'Q_2}{vr^3}, \quad h(r) = 1 - \frac{r_0^3}{r^3}, \quad (27)$$

and the dilaton is

$$e^{2\phi(r)} = \sqrt{\frac{(g\alpha'Q_2)^3}{v^3 r^6 (r^3 + g\alpha'Q_1)}}, \quad (28)$$

which vanishes in the limit $r \rightarrow \infty$. The temperature is

$$T \sim U_T^2 / \sqrt{g^2 Q_1 q_2}. \quad (\alpha'^2 U_0^3 \ll gQ_1) \quad (29)$$

A distance L between the quark and anti-quark is

$$\begin{aligned} L &= 2 \frac{1}{\alpha' \sqrt{gQ_1 v - gQ_2 U_0^4}} \int_1^\infty dx \sqrt{\frac{(\alpha'^2 x^3 U_0^3 + gQ_1)^2 gQ_2}{(x^3 - \lambda^3)(x^3 - 1)}} \\ &\rightarrow 2 \frac{gQ_1 \sqrt{gq_2}}{\sqrt{gQ_1 v - gQ_2 U_0^4}} \int_1^\infty dx \frac{1}{\sqrt{(x^3 - \lambda^3)(x^3 - 1)}}, \quad (\alpha' \rightarrow 0) \end{aligned} \quad (30)$$

where $x = U/U_0$ and $\lambda = U_T^3/U_0^3$. The energy is

$$E_{q\bar{q}} = \frac{1}{\pi\alpha'^2\sqrt{v(gQ_1v - gQ_2)U_0^4}} \int_1^\infty dx \sqrt{\frac{(\alpha'^2U_0^3 + gQ_1)(gQ_2)^2}{(x^3 - \lambda^3)(x^3 - 1)}} + \frac{U_T - U_0}{\pi}$$

$$\rightarrow \frac{gq_2\sqrt{gQ_1}}{\pi\sqrt{v(gQ_1v - gQ_2)U_0^4}} \int_1^\infty dx \frac{1}{\sqrt{(x^3 - \lambda^3)(x^3 - 1)}} + \frac{U_T - U_0}{\pi}, \quad (\alpha' \rightarrow 0) \quad (31)$$

where U_0 is the lowest value of U . We denote that the mass term of the W-boson is removed in the background. We take the limit $\lambda = U_T^3/U_0^3 \rightarrow 1$, the string tension is

$$T_{YM} = E_{q\bar{q}}/L \sim \sqrt{\frac{q_2}{4\pi^2Q_1}}. \quad (32)$$

The string tension is proportional to the squared mass of glueballs, namely

$$T_{YM} = g_{eff}M_{GB}^2, \quad (33)$$

where we define $g_{eff} = \sqrt{g^2Q_1q_2/4\pi^2U_T^2}$ as the effective coupling of the theory following [2,9,14]. Using this relation, glueball masses and KK masses are rewritten by

$$M_{GB} = \sqrt{\frac{U_T}{gQ_1}}, \quad T \sim \frac{U_T^2}{\sqrt{g^2Q_1q_2}} \sim \frac{U_T}{g_{eff}}, \quad (34)$$

and the string tension is rewritten by

$$T_{YM} = \sqrt{\frac{q_2}{4\pi^2Q_1}} = g_{eff}\frac{U_T}{gQ_1} \equiv c : \text{fixed}. \quad (35)$$

The curvature is

$$\alpha'R \sim \frac{r_0}{\sqrt{g^2Q_1Q_2}} = \frac{U_T}{\sqrt{g^2Q_1q_2}} \sim \frac{1}{g_{eff}}, \quad (36)$$

and the dilaton is rewritten by

$$e^\phi|_{U=U_T} = g_{eff}^{3/2} \frac{\sqrt{\alpha'}}{Q_1}. \quad (37)$$

The supergravity solutions describing D -branes can be trusted if the curvature in string units and the effective string coupling constant are small. Then we need to take the region

$$1 \ll g_{eff} \ll (Q_1^2/\alpha')^{1/3}. \quad (38)$$

In addition, in order to decouple Kaluza-Klein masses in the Euclidean time direction, we need to take the region

$$M_{GB} \ll M_{KK} \sim T. \quad (39)$$

Therefore we can obtain the QCD model that KK masses are decoupled without decoupling glueball masses in the region

$$g(Q_1\alpha')^{1/3}c \ll (gQ_1c^2)^{1/3} \ll U_T \ll gQ_1c. \quad (40)$$

V. CONCLUSION

We have studied the (3+1)-dimensional QCD model using the supergravity description of the non-extremal $D0$ - $D4$ branes without KK-modes in the Euclidean time direction. In the case of the supergravity background whose dilaton in the limit $r \rightarrow \infty$ is finite, we could not obtain the QCD model without decoupling KK-modes. However, in the case of the background whose dilaton in the limit $r \rightarrow \infty$ vanishes, we have found the region where KK modes in the Euclidean time direction are decoupled without decoupling glueball masses. The background has the boundary at $r \rightarrow 0$, which is needed in the supergravity - SYM correspondence. There are still some unwanted KK states that do not decouple. We have calculated the Wilson loops which exhibit the confining area law behavior, and the glueball mass spectrum is coincident with one in the non-extremal $D4$ -brane background. If we take into account dilaton fluctuations [27], there are some corrections due to the $D0$ -branes. We need to study the physical reasoning of the replacement of the harmonic function.

Comparing our results with that using rotating D -branes, discussed in [14–18], we share the results that the Yang-Mills tension is finite, and glueball masses are finite and small in the region that the curvature is small. The calculations of glueball masses and the string tension using the Wilson loop are more tractable in the our metric than that in the rotating D -branes. This is expected to be useful for further applications of our approach. For instance, Wilson loops of the baryon may be explicitly computed using our metric.

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